

I PRINCIPALI LIMITI NOTEVOLI

LIMITE DI NEPERO

$$\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

DERIVATI DAL LIMITE DI NEPERO

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1 \quad \Rightarrow \quad \ln(1+x) \underset{x \rightarrow 0}{\sim} x$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \quad \Rightarrow \quad e^x \underset{x \rightarrow 0}{\sim} 1 + x$$

$$\lim_{x \rightarrow 0} \frac{(1+x)^\alpha - 1}{x} = \alpha \quad \Rightarrow \quad (1+x)^\alpha \underset{x \rightarrow 0}{\sim} 1 + \alpha x$$

LIMITI GONIOMETRICI

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \Rightarrow \quad \sin x \underset{x \rightarrow 0}{\sim} x$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \quad \Rightarrow \quad \tan x \underset{x \rightarrow 0}{\sim} x$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \quad \Rightarrow \quad \cos x \underset{x \rightarrow 0}{\sim} 1 - \frac{x^2}{2}$$

Utile:

$$\lim_{x \rightarrow 0} x^\alpha \ln x = 0$$

SVILUPPI DI MC LAURIN

$$e^x = \sum_{n=0}^{+\infty} \frac{1}{n!} x^n = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + o(x^3)$$

$$\sin x = \sum_{n=0}^{+\infty} (-1)^n \frac{1}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^5)$$

$$\cos x = \sum_{n=0}^{+\infty} (-1)^n \frac{1}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + o(x^4)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + o(x^9)$$

$$\ln(1+x) = \sum_{n=1}^{+\infty} (-1)^{n+1} \frac{1}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$(1+x)^\alpha = \sum_{n=0}^{+\infty} \binom{\alpha}{n} x^n = \sum_{n=0}^{+\infty} \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!} x^n = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + o(x^3)$$

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} + o(x^5)$$

$$\arcsin x = x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{122} + \frac{35x^9}{1152} + o(x^9)$$

$$\arccos x = \frac{\pi}{2} x - \frac{x^3}{6} - \frac{3x^5}{40} - \frac{5x^7}{122} - \frac{35x^9}{1152} + o(x^9)$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + o(x^5)$$

$$\cosh x = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + o(x^5)$$

$$\sinh^{-1} x = x - \frac{x^3}{6} + \frac{3x^5}{40} - \frac{5x^7}{122} + \frac{35x^9}{1152} + o(x^9)$$